NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4053

THE COMBINATIONS OF THERMAL AND LOAD STRESSES FOR THE ONSET OF PERMANENT BUCKLING IN PLATES

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SUMMARY

A simple and practical method for evaluating the onset of permanent buckling in plates in the presence of combined thermal and compressive load stresses is outlined. A particular application of the method shows reasonable agreement with tests of 17-7 PH stainless-steel square tubes. The results indicate that the compressive load stress which the plate can support at the onset of permanent buckling is substantially reduced as the temperature difference of the plate and adjoining members increases.

INTRODUCTION

The use of high-density materials in supersonic aircraft causes concern with respect to buckling of the thin skin surfaces, particularly where permanent buckles may develop in the structure. Methods of analysis are needed which include the effects of thermal stresses in combination with the usual load stresses. In order to provide such methods, the behavior of plates due to combined thermal and load stresses is being studied; this paper presents initial results obtained for predicting the onset of permanent buckling and compares the results with experiment.

SYMBOLS

A_S cross-sectional area of skin, sq in.

A_W cross-sectional area of webs or stringers restraining thermal expansion of skin, sq in.

b width of skin, in.

e unit shortening of skin

E_S	modulus of elasticity of skin material at average skin temperature, ksi
E_W	modulus of elasticity of web material at average web temperature, ksi
t	thickness of skin, in.
ΔT	average temperature of skin minus average temperature of web, $^{\circ}F$
α	coefficient of thermal expansion, per $^{\circ}F$
ϵ_{cr}	strain at which buckling initiates
ϵ_{el}	elastic limit strain
σ	average stress over cross-sectional area of skin, ksi
σ_L	average compressive load stress over cross-sectional area of skin, ksi
σ_T	average thermal stress over cross-sectional area of skin, ksi

METHOD OF ANALYSIS

A simple approximation for the beginning of permanent buckling in a plate subjected to compressive load is suggested by the experimental observation that permanent buckling begins when the unit shortening of the plate is about the same value as the elastic-limit strain of the material. This concept appears useful for approximating the compressive load required for the onset of permanent buckling in a plate in the presence of thermal stresses. Consider a plate which has been shortened beyond the value required for buckling. (See fig. 1.) In the usual sense, the shortening of the plate comes from compressive loads; however, in effect, shortening also occurs when the plate is heated but the thermal expansion is restrained, such as would occur if the edges of the plate were bounded by stringers or shear webs at a lower temperature. This effective shortening due to restrained thermal expansion is the difference in length between the restrained and the unrestrained plate when heated. The lower curve of figure 2 shows the manner in which the effective unit shortening due to restrained thermal expansion increases with the difference in average temperature of the plate and the adjoining member. The dashed line in figure 2 indicates the critical strain or the unit shortening for buckling of the plate. The upper solid curve indicates the elastic-limit strain which decreases somewhat with increasing temperature. For a given temperature difference, then, the plate is in a state of shortening due to restrained thermal expansion; additional shortening by compressive loading causes the plate to buckle; and further compressive-load shortening causes the buckles to deepen but the buckles are not permanent until the limit given by the upper curve is exceeded. The region between the solid lines, then, defines the permissible amount of compressive-load shortening which may be applied in conjunction with the effective shortening from restrained thermal expansion without causing permanent buckling.

SPECIMENS AND METHOD OF TESTING

In order to test the validity of the foregoing approximate analysis, tests were performed on $\frac{1}{16}$-inch-thick 17-7 PH stainless-steel plates fabricated into square tubes by welding the corners. The tubes were 32 inches long and had b/t ratios of 40, 60, or 80. Two opposite walls of the tubes (skins) were exposed to heat supplied by two banks of quartz-tube radiators as shown in figure 3. The other two walls (webs) were shielded from the radiators by aluminum plates which ran lengthwise of the tube and projected diagonally outward from each corner of the tube. The shields have been removed in figure 3 in order to show the tube. Temperature distributions in the tubes were obtained with thermocouples, and records of the extension near the corners, along 15 inches of the length, were obtained with four differential transformers. Figure 3 shows the setup when the square tubes were subjected to heat without end load. The same test setup was placed in a testing machine and compressive loads were applied in combination with heat. In addition, compression tests of tubes at room temperature were performed. The square tubes were unloaded and/or cooled to room temperature after loading and/or heating, and profiles of the amplitude of the permanent buckles were obtained with a pantograph apparatus which multiplied the amplitude by a factor of 11. For each run, the average of the buckle depths (measured from crest to valley or twice maximum amplitude) in the skins was obtained. Figure 4 shows an example of the permanent-buckle information obtained for the square tubes subjected to heat. The particular data shown are for the square tube subjected to heat without compressive load and with $b/t = 60$. The ordinate shown in figure 4 is the difference of the average temperatures of the skin and web of the tube. Before the tube was subjected to heat, the average value of buckle depth was obtained from the pantograph measurements and is indicated in figure 4 (initial imperfection). After the tube had been subjected to an elevated temperature and cooled to room temperature, the permanent-buckle depth was slightly larger than the initial imperfection. Subsequent tests on the same tube but with progressively higher values of temperature resulted in progressively larger values of permanent-buckle depth. The temperature difference when the permanent-buckle depth began to exceed the initial imperfection was defined as the start of permanent buckling, as is indicated in figure 4. Similar information was obtained for all tubes tested except that load stress was plotted in place of temperature difference for the square tubes subjected to compressive load but not to heat. The load stress in the skin at the start of permanent buckling for the square tubes subjected to both heat and compressive load was evaluated from the extension measurements and the total load on the tube since the compressive stress in the skin for this case differs considerably from that in the webs.

RESULTS AND DISCUSSION

Figure 5 presents a comparison of the experimental results with curves obtained from the approximate analysis previously given. The ordinate shown in figure 5 is the average compressive load stress for the onset of permanent buckling in the plate, and the abscissa is the difference in average temperatures of the skin and web. The details of evaluation of the curves are given in the appendix. It is apparent that the compressive load stress which the plate can support at the onset of permanent buckling is substantially reduced as the temperature difference increases.

In general, reasonable agreement of the approximate analysis and the experiment exists. Somewhat better agreement exists when compressive load stress only is applied than when large thermal stresses are present. The poorer agreement is largely attributed to the assumption that the properties of the heated skin are represented by properties for the average value of skin temperature. This representation is somewhat inaccurate at high values of temperature difference when rather large-temperature gradients exist.

CONCLUDING REMARKS

A simple and practical method for evaluating the onset of permanent buckling in plates in the presence of combined thermal and compressive load stresses is outlined. A particular application of the method shows reasonable agreement with tests of 17-7 PH stainless-steel square tubes.

The results indicate that the compressive load stress which the plate can support at the onset of permanent buckling is substantially reduced as the temperature difference of the plate and adjoining members increases.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 6, 1957.

APPENDIX

EVALUATION OF COMPRESSIVE LOAD STRESS

Beyond buckling, the average-stress—unit-shortening relationship in the buckled skin of the square tubes was assumed to be (ref. 1)

$$\sigma = E_S\sqrt{e\epsilon_{cr}} \qquad \left(\epsilon_{cr} \leqq e \leqq \epsilon_{el}\right) \tag{1}$$

If it is assumed that permanent buckling begins when the unit shortening of the plate equals the elastic-limit strain, the combined thermal and compressive load stress for the onset of permanent buckling is

$$\sigma_T + \sigma_L = E_S\sqrt{\epsilon_{el}\epsilon_{cr}} \tag{2}$$

The values of E_S and ϵ_{el} for the various temperatures were obtained from compressive stress-strain tests of 17-7 PH stainless-steel sheet material. If uniform skin and web temperatures and complete continuity of the skin and webs are assumed, the thermal stress σ_T as obtained from elementary theory is

$$\sigma_T = \frac{\alpha E_S \ \Delta T}{1 + \dfrac{E_S A_S}{E_W A_W}} \tag{3}$$

Equation (3) was modified for thermal stresses exceeding the buckling stress by employing assumptions consistent with equation (1), in order to allow for the effect of the reduced longitudinal stiffness of the buckled skin. After evaluating the thermal stress, the compressive load stress σ_L was obtained from equation (2) and the results are shown by the curves of figure 5.

REFERENCE

1. Von Kármán, Theodor, Sechler, Ernest E., and Donnell, L. H.: The Strength of Thin Plates in Compression. A.S.M.E. Trans., APM-54-5, vol. 54, no. 2, Jan. 30, 1932, pp. 53-57.

BUCKLED PLATE

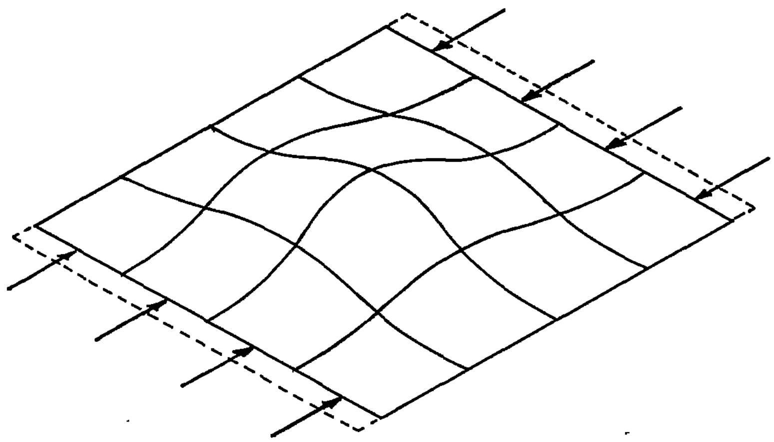

Figure 1

METHOD OF ANALYSIS

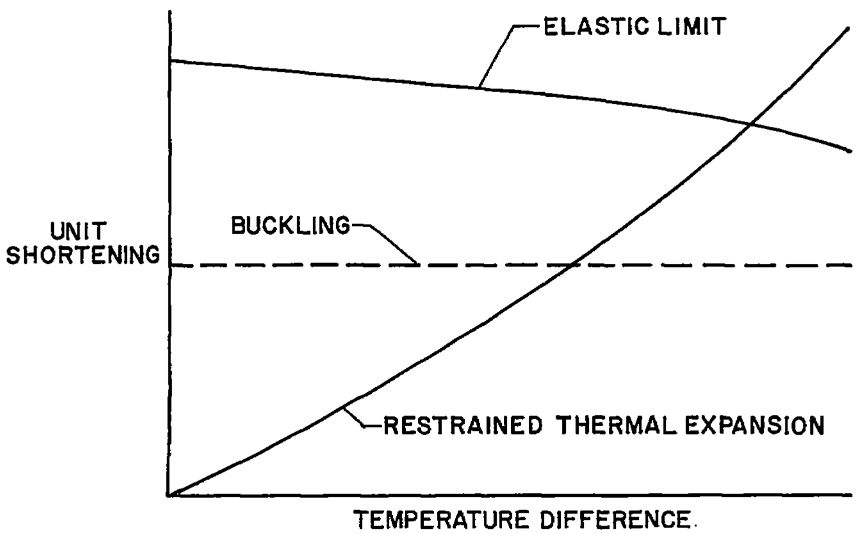

Figure 2

TEST SETUP FOR THERMAL PERMANENT BUCKLING
OF SQUARE TUBE

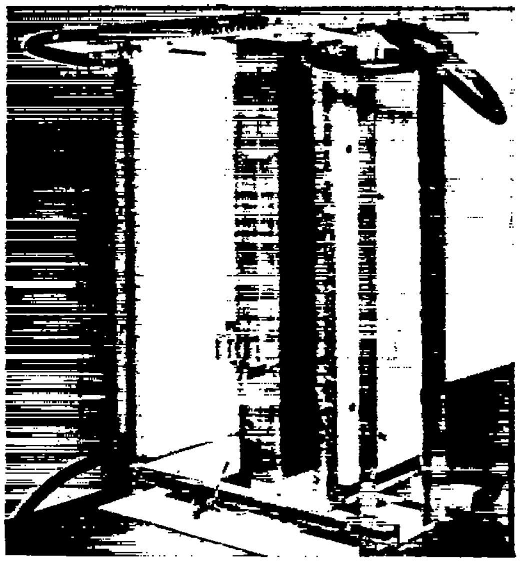

Figure 3 L-57-64.1

EXPERIMENTAL MEASUREMENTS OF PERMANENT-BUCKLE DEPTH
17-7 PH STAINLESS STEEL, b/t = 60

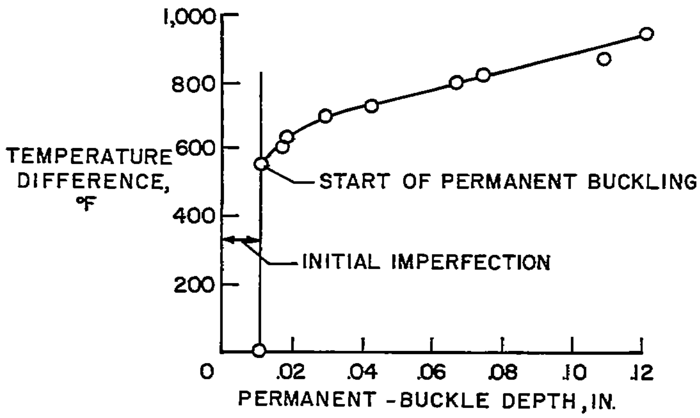

Figure 4

COMBINATIONS OF LOAD STRESS AND TEMPERATURE DIFFERENCE FOR THE START OF PERMANENT BUCKLING

17-7 PH STAINLESS STEEL; $A_S = A_W$

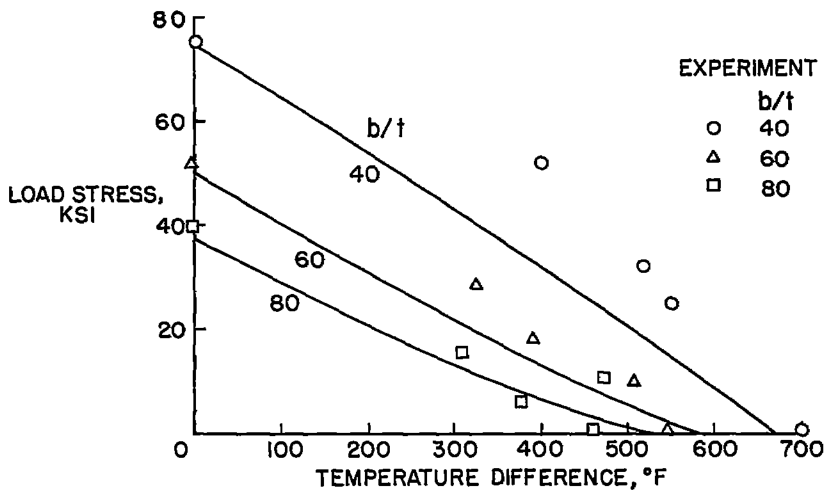

Figure 5